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EFFECTIVE INTERNAL LOSS FACTORS AND COUPLING LOSS FACTORS FOR NON-CONSERVATIVELY COUPLED SYSTEMS

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What causes negative loss factors in measuring SEA parameters by the power injection method? Analytical results in this paper show that negative loss factors are caused by non-conservative coupling, and non-conservative coupling does not always increase the effective internal loss factors (EILFs). To study the power flow of non-conservatively coupled systems, some new SEA models for non-conservatively coupled systems were suggested by researchers. However, the SEA parameters for non-conservatively coupled systems have not been extensively discussed by using the model of classical Statistical Energy Analysis (CSEA) up to now. The aim of this study is to give insight into the problem of the energy balance mechanism of non-conservatively coupled systems by using both the model and the energy balance equations in CSEA. A method of calculating EILFs and couplings loss factors (CLFs) for non-conservatively coupled machine structures is introduced. The possibility of causing negative loss factor is investigated by analysis of two non-conservatively coupled oscillators. Furthermore, the influences of the coupling stiffness and the coupling damping on EILFs and (CLFs) are investigated in detail. Finally, an application example is included to demonstrate the accuracy of the method.

1. INTRODUCTION

Statistical Energy Analysis (SEA) is an approach for analysis of random vibration of structures. In previous decades, it has been widely applied to the prediction of responses and control of mechanical noise and vibration. However, as Fahy [1] pointed out, strictly speaking, the energy balance equations for two coupled oscillators cannot be extended to more than two coupled oscillators because a simple condition of a linear chain of three oscillators, with external power supplied only to oscillator 1, indicates that the uncoupled modal energies of both oscillators 2 and 3 are zero, and hence no power will flow from oscillator 2 to oscillator 3. Therefore, it would cause serious errors to extend energy balance equations of two conservatively coupled oscillators randomly.

As one knows, in classical SEA, internal loss factor (ILF) represents the ability of a structure to dissipate its vibration energy; and coupling loss factor (CLF) represents the characteristic of energy transmission between physically coupled structures. However, neither ILF nor CLF includes the energy loss of the non-conservative coupling of the boundary.

For non-conservatively coupled systems, Chow and Pinnington [2] suggested that the loss of coupling damping of boundary be added to structural ILFs, so that the energy balance equations of conservatively coupled systems would be suitable for non-conservatively coupled systems. Recently, Beshara and Keane [3] introduced a new model and a

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new parameter named coupling damping loss factor for non-conservatively coupled systems showed that the coupling damping loss factor is an additional item which increases the ILFs of susbsystems.

Does non-conservative coupling always increase ILFs? Is there any influence of non-conservative coupling on CLFs? These questions attracted little attention till the negative ILFs were obtained by measurement using the power injection method [4–6]. In general, the non-conservative coupling affects both the ILFs and the CLFs. The loss of coupling damping is not proportionally added to the two coupled structures; and the CLFs are different from that of conservatively coupled systems. In what follows, the energy balance mechanism of non-conservatively coupled systems is investigated by using both the model and the enery balance equations in CSEA. To make a distinction from the ILFs of CSEA, the ILFs of non-conservatively coupled system are named the effective internal loss factors (EILFs). In the first half of this paper, the theoretical analysis of the ILFs and the CLFs of non-conservatively coupled structures are presented, and the engineering calculations of EILFs and CLFs are introduced; in the latter half, a model of two non-conservatively coupled oscillators is introduced to analyze the cause of negative EILFs, and the influence of non-conservative coupling on the ILFs and the CLFs are studied in detail. Finally, to demonstrate the accuracy of the method presented in this paper, an application example is included.

2. THEORETICAL ANALYSIS

Figure 1 shows the models of the two substructures system that have been analyzed. The energy balance equations of the substructures are

$$P_1 = P_{1d} + P_{12}^{\mathrm{I}} - P_{21}^{\mathrm{I}} = \omega_1\eta_1 E_1 + \omega_1\eta_{12}^{\mathrm{I}} E_1 - \omega_2\eta_{21}^{\mathrm{I}} E_2,$$

$$P_2 = P_{2d} + P_{21}^{\mathrm{II}} - P_{12}^{\mathrm{II}} = \omega_2\eta_2 E_2 + \omega_2\eta_{21}^{\mathrm{II}} E_2 - \omega_1\eta_{12}^{\mathrm{II}} E_1, \tag{1}$$

where P_{1d} and P_{2d} are powers dissipated by substructure 1 and 2 respectively, $P_{id} = \omega_i\eta_i E_i$ ($i = 1, 2$). η_i is determined by structural internal damping; E_i symbolizes the time-space averaged energy of substructure i; ω_i is the "blocked" frequency of oscillator i [7].

The energy transmissions that occurred in boundary I are, $P_{12}^{\mathrm{I}} = \omega_1\eta_{12}^{\mathrm{I}} E_1$ and $P_{21}^{\mathrm{I}} = \omega_2\eta_{21}^{\mathrm{I}} E_2$. Note that P_{12}^{I} is the power flow from substructure 1 to boundary I and P_{21}^{I} is the power flow from boundary I to substructure 1. Similarly, the energy transmissions that occur in boundary II are $P_{12}^{\mathrm{II}} = \omega_1\eta_{12}^{\mathrm{II}} E_1$ and $P_{21}^{\mathrm{II}} = \omega_2\eta_{21}^{\mathrm{II}} E_2$.

Figure 2 shows the SEA model of the non-conservatively coupled structures analyzed.

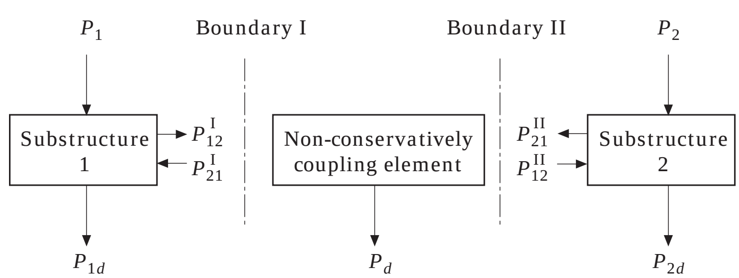

Figure 1. Model of two non-conservatively coupled structures.

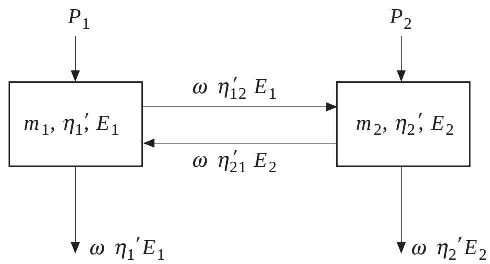

Figure 2. The SEA model of two non-conservatively coupled structures.

Using the *form* of energy balance equations of CSEA, and marking EILF as η_i', CLF as η_{ij}', the energy balance equations of non-conservatively coupled systems could be expressed as

$$P_1 = \omega_1\eta_1' E_1 + \omega_1\eta_{12}' E_1 - \omega_2\eta_{21}' E_2,$$
$$P_2 = \omega_2\eta_2' E_2 + \omega_2\eta_{21}' E_2 - \omega_1\eta_{12}' E_1, \tag{2}$$

Comparing equations (1) with equations (2) gives

$$\eta_{12}' = \eta_{12}^{\mathrm{II}}, \qquad \eta_{21}' = \eta_{21}^{\mathrm{I}}, \qquad \eta_1' = \eta_1 + (\eta_{12}^{\mathrm{I}} - \eta_{12}^{\mathrm{II}}), \qquad \eta_2' = \eta_2 + (\eta_{21}^{\mathrm{II}} - \eta_{21}^{\mathrm{I}}) \tag{3}$$

It can be found that η_{12}' and η_{21}' are just η_{12}^{II} and η_{21}^{I} respectively (see Figure 1), which is expected. η_1' includes the internal loss of substructure 1 and an item which symbolizes loss of non-conservative coupling. Similarly, η_2' is the sum of ILF of substructure 2 and non-conservative coupling loss $(\eta_{21}^{\mathrm{II}} - \eta_{21}^{\mathrm{I}})$. It can be proved that, for conservatively coupled systems, $\eta_{12}^{\mathrm{I}} - \eta_{12}^{\mathrm{II}} = \eta_{21}^{\mathrm{II}} - \eta_{21}^{\mathrm{I}} = 0$.

Since EILF is an important parameter in SEA, how to calculate EILF becomes the key of the problem. When substructures 1 and 2 are excited by the external force respectively, equations (2) are reduced to

$$\eta_{12}' = (\omega_2/\omega_1)E_{21}^{(1)}\eta_{s2}/(1 - E_{21}^{(1)}E_{12}^{(2)}), \qquad \eta_{21}' = (\omega_1/\omega_2)E_{12}^{(2)}\eta_{s1}/(1 - E_{21}^{(1)}E_{12}^{(2)}),$$
$$\eta_1' = (\omega_2/\omega_1)E_{21}^{(2)}\eta_{21}' - \eta_{12}', \qquad \eta_2' = (\omega_1/\omega_2)E_{12}^{(1)}\eta_{12}' - \eta_{21}', \tag{4}$$

where $E_{21}^{(1)}$ symbolizes the energy ration E_2/E_1 when only substructure 1 is excited. Similarly, $E_{12}^{(2)}$ symbolizes the energy ratio E_1/E_2 when only substructure 2 is excited. $\eta_{si} = P_i/\omega E_i$ $(i = 1, 2)$ is the total loss factor of coupled structure i. η_{si} and $E_{ij}^{(j)}$ can be measured by the power injection method [6]. The total loss factors and energy ratios could also be calculated as follows.

For a continuous structure, taking SEA parameters as the statistical average parameters in frquency band Δf of which the center frequency is f, then equations (4) become the expression of the CLFs and EILFs for non-conservatively coupled continuous structures. The energy ratio of the non-conservatively coupled continuous structures becomes the key to calculate the EILFs and the CLFs. Usually energy ratios can be expressed as [8],

$$E_{12}^{(2)} = \frac{4G_1G_2}{\omega n_2\eta_1}\frac{1}{|Y_1 + Y_2 + Y_c|^2}, \qquad E_{21}^{(1)} = \frac{4G_1G_2}{\omega n_1\eta_2}\frac{1}{|Y_1 + Y_2 + Y_c|^2}, \tag{5}$$

where G_1 and G_2, n_1 and n_2 are the averge admittances and modal densities of substructure 1 and 2 respectively. Y_1 and Y_2 are input point mobilities of substructure 1 and 2 respectively. Y_c is the mobility of coupling elements, $Y_c = \mathrm{j}\omega/\mathbf{K}$, and $\mathbf{K}$ is the complex stiffness of the coupling element [8]. The coupled structures are as shown in Figure 3.

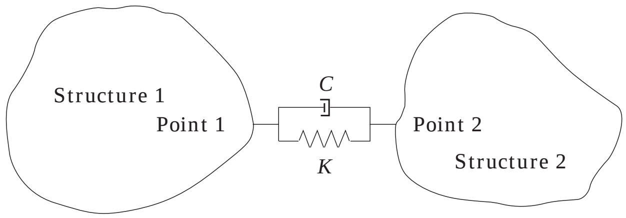

Figure 3. Two non-conservatively coupled machine structures.

Similarly, the total loss factors are deduced:

$$\eta_{s1} = \eta_1 \left| 1 - \frac{y_1^2}{Y_1(Y_1 + Y_2 + Y_c)} \right| \Big/ \left| 1 - \frac{y_1}{Y_1 + Y_2 + Y_c} \right|^2,$$

$$\eta_{s2} = \eta_2 \left| 1 - \frac{y_2^2}{Y_2(Y_1 + Y_2 + Y_c)} \right| \Big/ \left| 1 - \frac{y_2}{Y_1 + Y_2 + Y_c} \right|^2, \tag{6}$$

where, y_i is the transfer mobility of structure i, $i = 1, 2$. η_i is the ILF of structure i. The relationship between y_i and G_i were given by Lyon [9].

Substituting equations (5) and (6) into equations (4), the EILFs and CLFs of the non-conservatively coupled continuous structures could be obtained.

3. STUDY ON NON-CONSERVATIVELY COUPLED OSCILLATORS

It is difficult to analyze the influence of the coupling damping and the coupling stiffness on the EILFs and CLFs for continuous structures quantitatively. Therefore, a model of two non-conservatively coupled oscillators is introduced.

Figure 4 shows the analyzed two oscillators system. The vibration responses of the system are given by

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + c_3(\dot{x}_1 - \dot{x}_2) + k_1x_1 + k_3(x_1 - x_2) = F_1,$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + c_3(\dot{x}_2 - \dot{x}_1) + k_2x_2 + k_3(x_2 - x_1) = F_2, \tag{7}$$

where m_j, k_j, c_j symbolize the mass, stiffness and damping of oscillator j respectively.

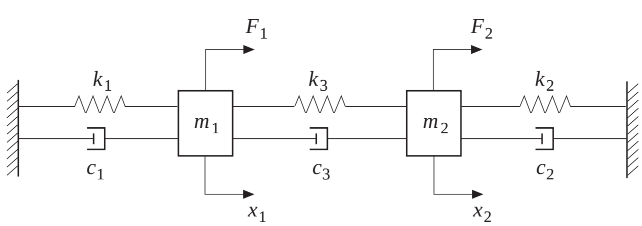

Figure 4. Model of two non-conservatively coupled oscillators.

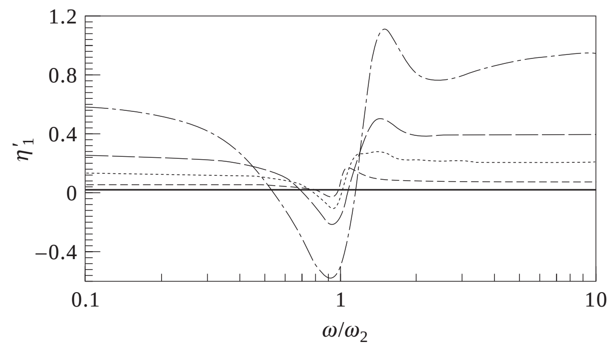

Figure 5. The influence of c_3 on η_1' (k_3 = 50 N/m): c_3 values; ——, 0·0; ---, 1·5; …., 5·0; — —, 10; –·–, 25.

In order for convenient expression, the following parameters are introduced:

$$\omega_1 = \sqrt{(k_1 + k_3)/m_1}, \qquad \omega_2 = \sqrt{(k_2 + k_3)/m_2}, \qquad \Delta_1 = \sqrt{(c_1 + c_3)/m_1},$$

$$\Delta_2 = (c_2 + c_3)/m_2,\ \mu = c_3/\sqrt{m_1 m_2}, \qquad v = k_3/\sqrt{m_1 m_2},$$

$$A = (\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) - \omega^2 \Delta_1 \Delta_2 - v^2 + \omega^2 \mu^2,$$

$$B = (\omega_1^2 - \omega^2)\Delta_2 + (\omega_2^2 - \omega^2)\Delta_1 - 2\mu v.$$

According to the definition of ILF,

$$\eta_1 = P_{1d}/\omega_1 E_1 = c_1/\omega_1 m_1, \qquad \eta_2 = P_{2d}/\omega_2 E_2 = c_2/\omega_2 m_2. \tag{8}$$

By considering that oscillators 1 and 2 are excited, respectively, it can be deduced that,

$$E_{12}^{(1)} = [(\omega_2^2 - \omega^2)^2 + \omega^2 \Delta_2^2]/(v^2 + \omega^2 \mu^2), \qquad E_{21}^{(2)} = [(\omega_1^2 - \omega^2)^2 + \omega^2 \Delta_1^2]/(v^2 + \omega^2 \mu^2), \tag{9}$$

$$\eta_{12}' = \frac{1}{\omega}\frac{\omega_2}{\omega_1}\frac{(v^2 + \omega^2\mu^2)[(\omega_1^2 - \omega^2)B - \Delta_1 A]}{[(\omega_1^2 - \omega^2)^2 + \omega^2\Delta_1^2][(\omega_2^2 - \omega^2)^2 + \omega^2\Delta_2^2] - (v^2 + \omega^2\mu^2)^2} \tag{10.1}$$

$$\eta_{21}' = \frac{1}{\omega}\frac{\omega_1}{\omega_2}\frac{(v^2 + \omega^2\mu^2)[(\omega_2^2 - \omega^2)B - \Delta_2 A]}{[(\omega_1^2 - \omega^2)^2 + \omega^2\Delta_1^2][(\omega_2^2 - \omega^2)^2 + \omega^2\Delta_2^2] - (v^2 + \omega^2\mu^2)^2} \tag{10.2}$$

Substituting equtions (9) and equations (10) into equations (4), EILFs are obtained.

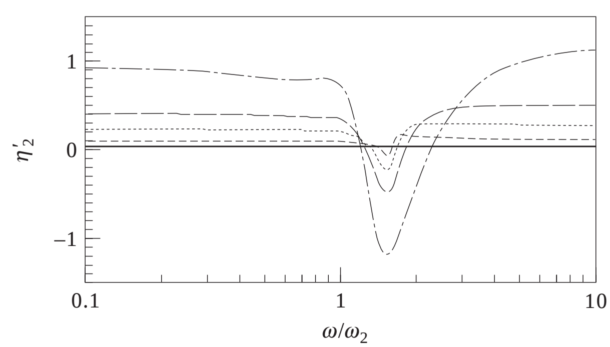

Figure 6. The influence of c_3 on η_2' (k_3 = 50 N/m). Key as for Figure 5.

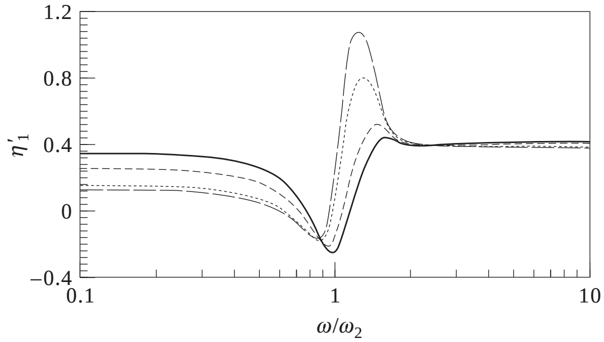

Figure 7. The influence of k_3 on η_1' ($c_3 = 10$ Ns/m): k_3 values; ——, 20; ---, 50; ····, 100; — —, 120.

It is still difficult to analyze the influence of the coupling stiffness and the coupling damping on the EILFs and CLFs directly from equations (10) theoretically. Therefore numerical calculations have to be carried out. In the following analysis, parameters of oscillators are fixed. That is, $m_1 = 1{\cdot}5$ kg, $k_1 = 400$ N/m, $c_1 = 0{\cdot}5$ Ns/m, $m_2 = 2$ kg, $k_2 = 200$ N/m, $c_2 = 1{\cdot}0$ Ns/m.

Figures 5 and 6 show the influence of the coupling damping on the EILFs. Affected by the non-conservative coupling, η_1' varies greatly near ω_2. When the driving frequency is a little lower than ω_2, η_1' shows a valley. As the driving frequency increases, the valley becomes obvious, and even assumes negative values; when the driving frequency is a little higher than ω_2, η_1' shows a peak, and the greater the damping, the higher the peak, until it exceeds 1. The curve of η_2' is similar to the curve of η_1', but the valley of η_2' corresponds to the peak of η_1'. The law of the EILFs changes with the driving frequency is: the larger the coupling damping, the greater the EILFs change, and the easier for the EILFs to be grater than 1 or smaller than 0. As expected, in the range far away from resonance frequencies, the EILFs increase as the driving frequency increases. Furthermore, contrasting Figure 5 with Figure 6, it is shown that η_1' and η_2' would not both be negative at the same time. However, they will both be greater than 1 if the coupling damping is large enough. In fact, because of the use of the *form* of the energy balance equations of conservatively coupled systems for the non-conservatively coupled ones, it is an inevitable result of the energy balance.

Figures 7 and 8 show the influence of the coupling stiffness on the EILFs. It is shown that the larger the coupling damping, the greater the EILFs drop from peak to valley, which illustrates that the function of coupling stiffness to the change of the EILFs can be

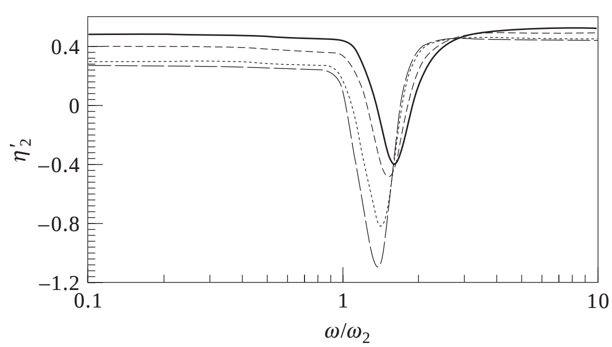

Figure 8. The influence of k_3 on η_2' ($c_3 = 10$ Ns/m). Key as for Figure 7.

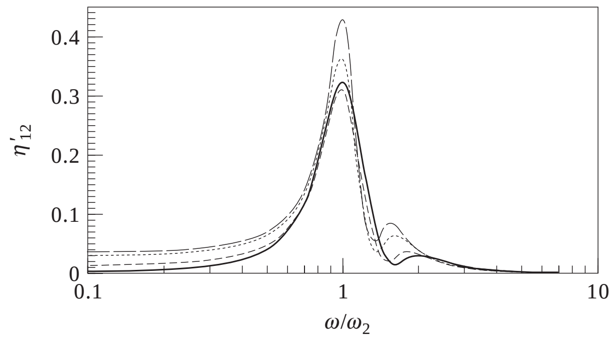

Figure 9. The influence of k_3 on η'_{12} ($c_3 = 10$ Ns/m). Key as for Figure 7.

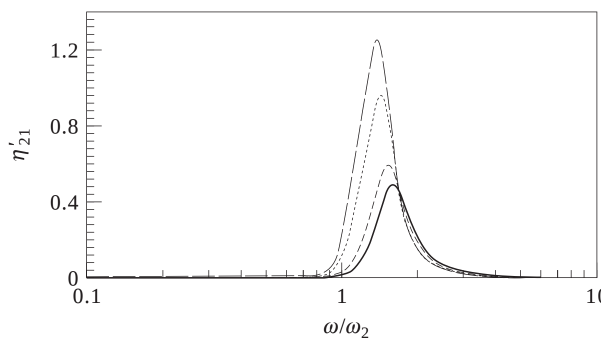

Figure 10. The influence of k_3 on η'_{21} ($c_3 = 10$ Ns/m). Key as for Figure 7.

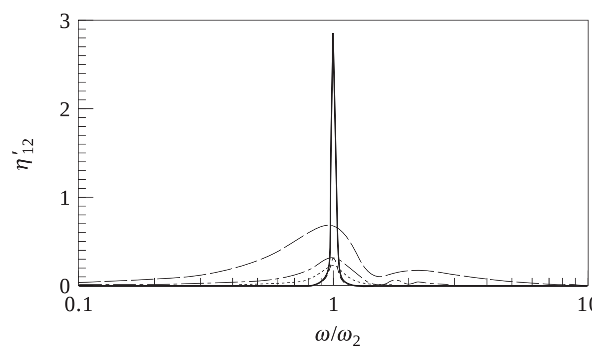

Figure 11. The influence of c_3 on η'_{12} ($k_3 = 50$ N/m). Key as for Figure 5

considered as "catalyzing". "Catalyzing" becomes more serious when the coupling stiffness increases. Furthermore, the valley of η'_2 corresponds to the peak of η'_1.

Figures 9 and 10 show the influence of the coupling stiffness on CILFs. It is shown that the maximum CLF occurs in the resonance frequency of the reaching oscillator, and energy transmission is relatively critical around the resonance frequencies of the coupled oscillators. This law is familiar to the authors [10].

Little is known of the influences of the coupling damping on the CLFs, which are shown in Figures 11 and 12. The curves show sharp peaks in the resonance frequencies of the

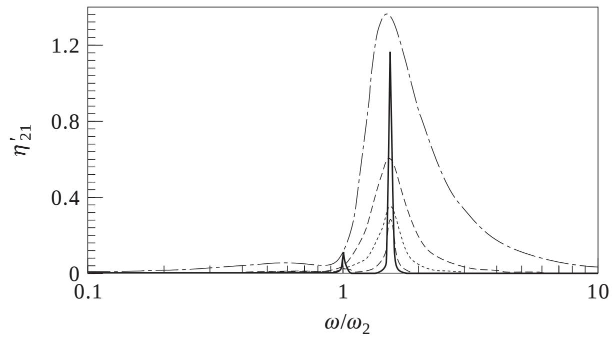

Figure 12. The influence of c_3 on η'_{21} ($k_3 = 50$ N/m). Key as for Figure 7.

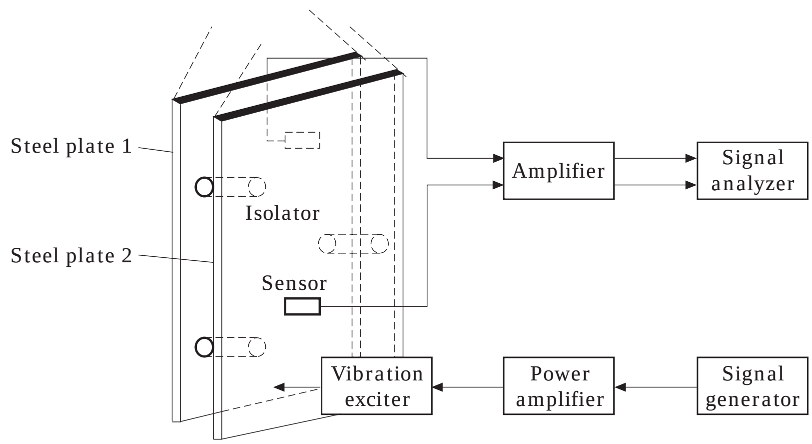

Figure 13. Experimental system.

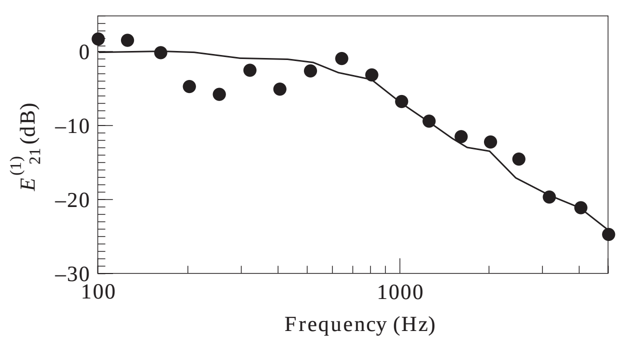

Figure 14. Variation in the energy ratio of plates with frequency: —, predicted; ●, measured.

oscillators when the two oscillators are conservatively coupled. It illustrates that the coupled oscillators absorbs a lot of energy from the physically linked oscillator at its resonance frequency. This law is still true for non-conservative coupling, but the peak is smoother. It is also found that the greater the coupling damping, the bigger the CLFs, which means that the function of the coupling damping in the increase of the CLFs can also be considered as "catalyzing".

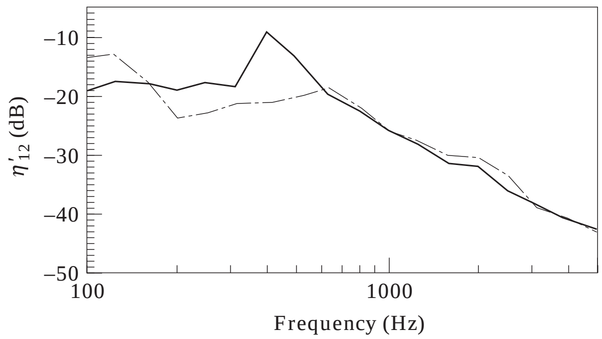

Figure 15. Variation in CLF with frequency. ——, predicted; –·–, measured.

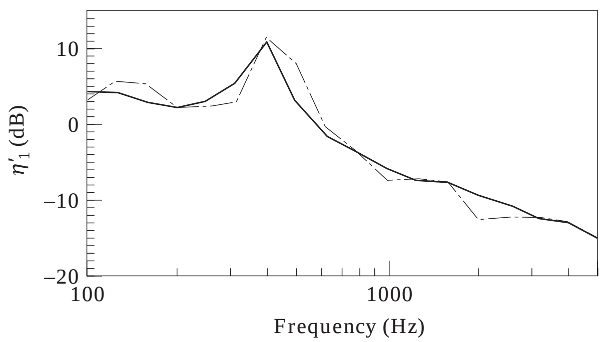

Figure 16. Variation in EILF with frequency. Key as for Figure 15.

4. A REAL EXAMPLE

To verify this the approach to the evaluation of the energy ratios, the EILFs and the CLFs for non-conservatively coupled structures, experimental data are obtained and compared with the predicted results.

The experimental system is made of two steel plates, the thicknesses of which are 2 mm and 4 mm, and both plates are 500 mm × 500 mm. Three isolators are stuck between the two plates. The plates are slightly damped with damping material to increase the ILFs. Thus, high precision is possible in measuring the ILFs. The experimental system is as shown in Figure 13. Only flexural wave is considered in this model.

The measured and calculated energy ratios are compared in Figure 14. It is found that the measured results agree well with the theoretical data, even in low frequency.

Figures 15 and 16 give curves of η'_{12} and η'_1 respectively. The tested CLF agrees with the calculated CLF for the middle and high frequencies. There are some differences between the measured and predicted CLFs at low frequencies, which is acceptable in SEA. The measured and predicted ILFs also have an identical tendency.

5. CONCLUSIONS

The theoretical and numerical analyses of the two non-conservatively coupled oscillators illustrate that it should no longer be unexpected to obtain abnormal values of ILFs by the power injection method. Combining the analysis of the two non-conservatively coupled oscillators, and the real example provided, the following conclusions can be drawn:

(1) The EILFs of the non-conservatively coupled system include not only structural internal losses, but also loss of the coupling. For conservatively coupled system, the EILFs equal the ILFs.

(2) The peaks and valleys of the EILFs and the CLFs are formed when the driving frequency is near to the resonance frequency.

(3) The peaks and valleys of the EILFs and the CLFs become obvious when the coupling damping increases, and it is possible for EILFs to be greater than 1 or smaller than 0; when the driving frequency is far away from the resonance frequencies, the EILFs increase as the coupling damping increases.

(4) The coupling stiffness has a "catalyzing" function in the change of the EILFs. "Catalyzing" is serious when the coupling stiffness increases.

(5) The coupling damping acts as "catalyzing" to the change of the CLFs. "Catalyzing" is serious when the coupling damping increases.

(6) The accuracy of the method for engineering applications is demonstrated by a real example.

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